# Predicate Logic: Symbols and Translations

So far, we have discussed two different versions of deductive logic: **categorical logic** (which deals with relationships between categories) and **propositional logic** (which deals with relationships between statements/propositions). These two forms of logic are sufficient to establish the validity of many deductive arguments. They are also very old—categorical logic goes back to Aristotle (~300 BCE), while propositional logic goes back to the Greek and Roman Stoics (~200 BCE). For around 2,000 years, these were the only versions of deductive logic around, and many people thought that logic had been “completed.”

The final version of deductive logic will be studying, called **first-order logic** (or **predicate logic**) was developed in the late 1800s and early 1900s to deal with issues that arose in mathematical proofs. Predicate logic now plays a key role in helping to answer questions that come up in philosophy, linguistics, mathematics, and computer science (most of the first computer scientists were experts in first order logic). Predicate logic is far more powerful than either categorical or propositional logic, and it can account for the validity of all of the arguments that these logics can account for, plus many more. Most professional logicians (including mathematicians, philosophers, and computer scientists) spend most of their time using predicate logic.

## The basics of Predicate Logic: Predicates, Constants, Quantifiers, and Variables

The simplest sort of claim in predicate logic involves two different things: a capital letter symbolizing a **predicate** (or “property”) and an **individual constant** (or particular object, person, place, etc.). So, for example, here are some sample predicates and constants:

|  |  |  |
| --- | --- | --- |
| Expression | What is it? | Meaning |
| a | Individual constant | Alice |
| q | Individual constant | The Queen of Hearts |
| W\_\_ | Predicate | \_\_\_\_ is in wonderland |
| M\_\_ | Predicate | \_\_\_\_ is mad |
| Wa | Simple proposition | Alice is in wonderland |
| Mq | Simple proposition | The Queen is mad |
| Ma ∨ (Wa ∙ Mq) | Compound proposition | Alice is mad, or Alice is in wonderland and the queen is mad. |
| ~Wq ≡ Wa | Compound proposition | The queen is not in wonderland if and only if Alice is in wonderland. |

As you can see, predicate logic allows you to use all of the **logical operators** from propositional logic (∨, ∙, ≡, ⊃, ~).

Predicate logic also has **quantifiers** and **variables.** A **universal quantifier** (“All” in categorical logic) is written as (x), while the **existential** quantifier (“Some” in categorical logic) is written as ∃. Every quantifier is *always* accompanied by a **bound** variable (represented by x, y, or z) showing what the quantifier is “ranging” over. (A variable takes the place of an individual constant.) If an expression of predicate logic contains a **free variable** (one not bound by a quantifier), it is not a complete sentence. Using our previous example, here are some examples of claims we could make in predicate logic.

|  |  |  |
| --- | --- | --- |
| Expression | What is it? | Meaning |
| x | Free Variable | Nothing (by itself) |
| Wx | Incomplete (free variable) | x is in wonderland (incomplete sentence, since x doesn’t refer to anything) |
| (x)Wx | Proposition | Every single thing is in wonderland |
| (x)(Wx ⊃ Mx) | Proposition | If something is in wonderland, that thing is mad. |
| (x)Wx ⊃ (x)Mx | Proposition | If everything is in wonderland, everything is mad. |
| (x)Wx ⊃ Mx | Incomplete (free variable) | If everything is in wonderland, then x is mad (incomplete sentence, since x doesn’t refer to anything) |
| ∃xWx | Proposition | There exists at least one thing in wonderland. |
| ∃xWx ∙ ∃xMx | Proposition | There exists at least one thing in wonderland and one thing that is mad. |
| ∃x(Wx ∙ Mx) | Proposition | There exists at least one thing in wonderland and that thing is mad. |

It’s important to note that quantifier only “applies to” or “governs” the statement directly next to it. For example, (x)Wx ⊃ (x)Mx and (x)(Wx ⊃ Mx) mean very different things.

## Translating Statements into Predicate Logic

Using the concepts laid out above, we can now translate a wide variety of claims into predicate logic. In order to keep matters simple, we won’t be dealing with relationships (“Bill is friends with Beatrice”) or identity (“Samuel Clemens is the same person as Mark Twain”).

|  |  |
| --- | --- |
| Statement | Translation |
| Garfield is a cat | Cg |
| Snoopy is not a cat | ~Cs |
| All cats are mammals / Every cat is a mammal. | (x)(Cx ⊃ Mx) |
| No dogs are cats | (x)(Dx ⊃ ~Cx) |
| Some birds can swim | ∃x(Bx ∙ Sx) |
| Some politicians are not honest. | ∃x(Px ∙ ~Hx) |
| Dragons do not exist | (x)(~Dx) |
| Gophers do exist (There exists at least one gopher) | ∃xGx |
| No children like needles | (x)(Cx ⊃ ~Nx) |
| Frannie will buy a coat only if George does. | Cf ⊃ Cg |
| Frannie will buy a coat if George does. | Cg ⊃ Cf |
| A few philosophers are logicians. | ∃x (Px ∙ Bx) |
| It is not the case all philosophers are logicians. | ~(x)(Px ⊃ Lx) OR ∃x(Px ∙ ~Lx) |

## Review Questions

Translate the following into predicate logic. The individual constants and predicates are noted in parentheses:

1. Judith is a philosopher (j, P).
2. LeBron is not a football player (l, F).
3. Neither Aristotle nor Descartes spoke English (a, d, E).
4. Both Hermione and Ron are wizards (h, r, W).
5. My father is either an accountant or a truck driver (f, A, T).
6. All potatoes are vegetables (P, V).
7. No bananas are vegetables (B, V).
8. Some restaurants do not have good service (R, G).
9. Some phones are made by apple (P, A).
10. If Jones is hot, he is wearing shorts (j, H, S).
11. Good shoes are expensive. (G, S, E)
12. Beth and Carl are both college students (b, c, C).
13. The Vikings are either a football team or a basketball team (v, F, B).
14. Some Italian restaurants are not well-reviewed (I, R, W).
15. If all cars are expensive, then Joe’s car is expensive (C, E, j).